Growth in a Differentiated Oligopoly with Product Innovation[1]

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Abstract

We present a two-sector model where oligopolistic firms sell differentiated products. New products are introduced over time through formal R&D activity. Market competition takes place either *à la* Cournot or *à la* Bertrand. We show that tougher competition associated with price-setting behaviour does not entail a larger social welfare as long as it requires more effort for the production of the final goods.

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1 Introduction

We present a model of oligopolistic competition and horizontal product innovation through research activity. The novelty of the model stands in bridging the literature on oligopolistic competition (see Singh and Vives, 1984; Vives, 1985; Okuguchi, 1987, *inter alia*), with the analysis of endogenous creation of new varieties developed in the literature on endogenous growth (see Lucas, 1988; Grossman and Helpman, 1991). We model market competition alternatively *à la* Bertrand and *à la* Cournot. Our prominent result is that the tougher competition associated with price-setting behaviour (compared to quantity setting behaviour) does not entail a larger social welfare, as long as it requires more effort in the production of physical goods, and less resources available for R&D.

In the existing literature on oligopoly markets, welfare evaluations between price and quantity competition are usually carried out for a *given* market structure, that is, for a given number of firms (Vives, 1985; Okuguchi, 1987; Cellini and Lambertini, 1998). For a given number of firms (and product varieties), the harsher competition characterising a Bertrand behaviour *vis à vis* its Cournot counterpart implies that price competition is socially preferable to quantity competition.

The aim of our paper can be outlined as follows. We want to endogenise the equilibrium market structure, given two alternative forms of market competition, i.e., Bertrand and Cournot. The number of firms in the long-run equilibrium is determined by the amount of resources available for R&D activity. In particular, we prove that Cournot competition allows for a larger number of firms to enter. Hence, the conventional wisdom on industry output and social welfare may not hold. In fact, we show that there exists a region of parameters for which the economic system converges to a steady state where social surplus is larger under Cournot than under Bertrand competition.

To our knowledge, this result has been overlooked so far in the literature, possibly because the issue of product diversity has been mostly tackled in terms of monopolistic competition rather than oligopoly. As it is well known (see, e.g. Dixit and Stiglitz, 1977), under monopolistic competition the choice between setting prices and setting quantities is irrelevant for firms.[1]

The remainder of the paper is structured as follows. The model is laid out in section 2. Dynamic analysis is in section 3. Section 4 focuses upon

[1] Several models of endogenous growth with monopolistic competition are available (Grossman and Helpman, 1991; Aghion and Howitt, 1992; Galì, 1994, 1996, *inter alia*). On the contrary, the literature on endogenous growth and oligopoly is very limited (Peretto, 1996, 1998; Vencatachellum, 1998).

the welfare properties of the steady states. Discussion and conclusions are gathered in section 5.

2 The setup

We consider a three-sector economy with only one factor of production. The first two sectors produce final goods. One sector is competitive and supplies the numeraire good, the other is an oligopoly market where single-product firms sell differentiated products. The third sector is competitive and carries out the R&D activity aimed at product innovation. The only production factor is labour and total labour force available in the economy is L. We solve the model by backward induction. We start by characterising the market equilibrium for a given number of firms. Then, we investigate how the R&D activity endogenously determines the number of firms.

2.1 The differentiated goods market

The demand structure is borrowed from Spence (1976). The market is supplied by n firms. The inverse demand function for variety i is:

$$p_i = a - bq_i - d\sum_{j\neq i} q_j \tag{1}$$

where $d \in [0, b]$ is the symmetric degree of substitutability between any pair of varieties. If $d = b$, products are completely homogeneous; if $d = 0$, products are completely independent and each firm becomes a monopolist. The corresponding direct demand function for variety i is (Majerus, 1988):

$$q_i = \frac{1}{b + d(n-1)} \cdot \left\{ a - \frac{b + d(n-2)}{b-d} \cdot p_i + \frac{d}{b-d} \cdot \sum_{j \neq i} p_j \right\} . \qquad (2)$$

Duality and integrability theorems allow to derive the indirect utility function corresponding to (1) and (2) (see Irmen, 1997):

$$V = -\frac{a}{b+d} \cdot \sum_i p_i + \frac{b}{2(b^2 - d^2)} \cdot \sum_i (p_i)^2 - \frac{d}{b^2 - d^2} \cdot \sum_i \sum_{j \neq i} p_i \cdot p_j + E \ , \quad (3)$$

where E is individual expenditures for the consumption of the differentiated good and the competitive numeraire good. We assume that the numeraire good is supplied under constant returns to scale and we set its unit input coefficient to one by choice of units so that in equilibrium the wage rate is also equal to 1.

Production of differentiated goods takes place through the following technology:

$$q_i = \frac{1}{\alpha} l_i \qquad (4)$$

where l_i denotes the amount of labour employed in the production of variety i, and $1/\alpha$ is the constant average productivity of labour. Total costs borne by firm i are $TC_i = \alpha q_i$.

Firm i's objective function is $\pi_i^k = p_i q_i - TC_i$. Superscript $k = B, C$ indicates whether market competition takes place *à la* Cournot or *à la* Bertrand.

When firms compete in quantities, firm i's profit function is:

$$\pi_i^C = \left(a - bq_i - d \sum_{j \neq i} q_j - \alpha \right) q_i \ . \tag{5}$$

The first order condition for firm i is:

$$\frac{\partial \pi_i^C}{\partial q_i} = a - \alpha - 2bq_i - d \sum_{j \neq i} q_j = 0 \ . \tag{6}$$

From (6) we immediately derive the best reply function:

$$q_i = \frac{a - \alpha - d \sum_{j \neq i} q_j}{2b} \ . \tag{7}$$

On the basis of ex ante symmetry across the population of firms, we introduce the following assumption:

$$\sum_{j \neq i} q_j = (n-1) q_i \ , \tag{8}$$

thanks to which we can drop, in the remainder, the indication of the identity of the firm. The individual output level in equilibrium is

$$q^C(n) = \frac{a - \alpha}{2b + d(n-1)} \ , \tag{9}$$

to which the following profits are associated:

$$\pi^C(n) = \frac{(a-\alpha)^2 b}{[2b+d(n-1)]^2} \; . \tag{10}$$

An obvious non-negativity constraint on individual output (9) is $a \geq \alpha$. Industry output is $Q^C(n) = nq^C(n)$. The overall amount of labour employed by the industry in equilibrium, for the production of differentiated varieties, is $L_F^C = \alpha Q^C(n) \in (0, L]$, where subscript F denotes that this amount of labour is employed to produce the final goods.

Under price competition, the individual profit function is:

$$\pi_i^B = \frac{p_i - \alpha}{b+d(n-1)} \cdot \left\{ a - \frac{b+d(n-2)}{b-d} \cdot p_i + \frac{d}{b-d} \cdot \sum_{j \neq i} p_j \right\} \; . \tag{11}$$

Calculating the first order condition on (11) w.r.t. p_i, and then using the symmetry assumption $\sum_{j \neq i} p_j = (n-1)p$, we derive the equilibrium price

$$p^B(n) = \frac{a(b-d) + \alpha\,[b+d(n-2)]}{2(b-d)+d(n-1)} \; . \tag{12}$$

Plugging (12) into the profit function (11) and simplifying, we get the individual equilibrium profits:

$$\pi^B(n) = \frac{(a-\alpha)^2 (b-d)\,[b+d(n-2)]}{[2(b-d)+d(n-1)]^2\,[b+d(n-1)]} \; . \tag{13}$$

Individual and industry output are, respectively,

$$q^B(n) = \frac{(a-\alpha)\,[b+d(n-2)]}{[2(b-d)+d(n-1)]\,[b+d(n-1)]} \; , \tag{14}$$

and $Q^B(n) = nq^B(n)$, requiring the employment of an overall amount of labour $L_F^B = \alpha Q^B(n) \in (0, L]$.

2.2 The R&D sector

The activity carried out in the R&D industry is summarised by the following production function:

$$\dot{n} \equiv \frac{dn}{dt} = \frac{1}{\beta} L_R \,, \tag{15}$$

where β is a positive parameter, and $L_R \in [0, L)$ is the amount of labour employed in this sector.[2] Subscript R stands for *research and development*. To simplify notation, we omit the indication of time in (15). The following remarks are in order:

- Technology (15) is a special case of a commonly used R&D function, namely, $\dot{n} = L_R n^\gamma / \beta$, with the restriction $\gamma = 0$. This restriction implies that

- There exists no learning by doing. In other words, the number of existing varieties does not affect labour productivity in the R&D sector.

- The R&D technology exhibits constant returns to scale.

[2] >From now on we abstract from the integer problem and treat n as a real number.

- As a result, we know that the dynamic system converges to a steady state (Solow, 1992, 1994).

Resources are devoted to R&D according to intertemporal saving decisions by consumers, who lend to firms at the market interest rate r. Intertemporal utility is log-linear with rate of time preference ρ, which implies the Euler equation:

$$\frac{\dot{E}}{E} = r - \rho \tag{16}$$

where E is individual expenditures in current consumption.

Since the wage is constant and no learning by doing is observed, the cost for introducing a new variety is also constant and equal to β. The value of a new variety must correspond to the discounted flow of profits generated by the same variety. Under perfect competition in the R&D sector, this entails:

$$\Pi_{i,s}^{k} \equiv \int_{s}^{\infty} e^{-rt} \pi_{it}^{k} dt = \beta \ , \tag{17}$$

As β is constant over time, the present value of profits $\Pi_{i,s}$ associated with variety i introduced at time s, must be constant as well, and the Fischer equation simplifies as follows:

$$\frac{\pi^{k}(n)}{\beta} = r \ , \tag{18}$$

which can be substituted into (16) to give:

$$\frac{\dot{E}}{E} = \frac{\pi^k(n)}{\beta} - \rho \tag{19}$$

3 Dynamics

By (15), the dynamics of n may be written as $\dot{n} = (L - L_F - L_N)/\beta$, where L_N is the amount of labour employed in the production of the numeraire. We already know that $L_F = \alpha Q(n)$ and we can readily determine L_N by considering the corresponding product market clearing condition: $L_N = E \cdot L - p(n)Q(n)$ so that:

$$\dot{n} = [L + \pi^k(n)n - E \cdot L]/\beta \ , \quad n_0 > 0 \tag{20}$$

which shows that innovation is fed by saved income $[L + \pi^k(n)n - E \cdot L]$.

Together (19) and (20) form a two-dimensional system of differential equations that mirrors the standard Ramsey system.

When $\dot{E} = 0$ and $\dot{n} = 0$, the system is in steady state. To find the corresponding number of varieties is a matter of straightforward calculations First, imposing $\dot{E} = 0$ in (19) defines the steady state number of firms n^k_{ss} as the solution to:[3]

$$\pi^k(n) = \rho\beta \tag{21}$$

Then, substituting (21) and n_{ss}^k in (20), condition $\dot{n}= 0$ gives steady state expenditures:

$$E_{ss}^k = (1 + \rho\beta n_{ss}^k)/L \tag{22}$$

where, since β is the value of a variety (see (17)), $\rho\beta n_{ss}^k$ represents the annuity value of the steady state stock of varieties.

The relevant parameter space is $\Lambda = \{a, b, d, L, \alpha, \beta, \rho\}$. Numerical simulations over Λ indicate that the standard result on quantities, namely $q_{ss}^B > q_{ss}^C$, holds so that price competition leads to larger per-firm output of the differentiated good than quantity competition. However the associated inequality on the numbers of varieties is $n_{ss}^B < n_{ss}^C$. In plain words, the dynamic evolution of the system produces a steady state where more varieties exist under Cournot behaviour than under Bertrand behaviour. The intuition is straightforward: since price competition is harsher than quantity competition, the former requires more resources than the latter for production, limiting thus the ability of the innovative sector to introduce new varieties.

[3] The expression of n_{ss}^C is in the appendix. The expression of n_{ss}^B is available upon request.

Therefore, it is not a priori obvious that price competition is superior to quantity competition from a welfare point of view.

4 Welfare in steady state

To obtain the social welfare level in the two steady states associated with Bertrand and Cournot competition, we substitute the relevant steady state magnitudes into the indirect utility(3).

Notice that the inequality $n_{ss}^B < n_{ss}^C$ might imply $V_{ss}^B < V_{ss}^C$, that is, contrary to the established wisdom, the indirect utility of the representative consumer in steady state may be higher under Cournot competition than under Bertrand competition. Intuitively, price competition is harsher than quantity competition. Hence, the former absorbs a larger amount of labour than the latter for the production of the final goods. Thus, under Bertrand behaviour, a smaller amount of resources is available for the innovative activity. The ultimate implication is that Bertrand behaviour may yield a higher degree of concentration in the market for the differentiated goods, damaging consumers. Moreover, notice that, due to the fact that we carry out a general equilibrium analysis, expenditures E_{ss}^k depend positively on the steady state number of firms (which exerts another effect in favor of Cournot).

Consider now industry profits in the two settings. Given that per-firm profits are the same under both Cournot and Bertrand behaviour (see (21)), industry profits are larger under Cournot competition because $n_{ss}^C > n_{ss}^B$. This has some relevant implications as to firms' preferences concerning the choice of the market variable. The established wisdom states that quantity-setting behaviour is a dominant strategy for firms (Singh and Vives, 1984). In the present setting, the individual firm has no such preference, on the basis of (21). Nevertheless, *ex ante* (i.e., before the entry process starts), every firm prefers Cournot behaviour since it allows for a larger number of firms to survive in the long run equilibrium.

The overall appraisal of social welfare in the two settings remains to be carried out. On the basis of the foregoing discussion, there may exist cases where Cournot is socially preferred to Bertrand. An extreme situation where this is indeed the case obtains if varieties are very close substitutes, i.e., $d \to b$. If so, then price competition yields a steady state where the market for the final good is a monopoly, because entry by any other firm would drive individual instantaneous profits to $\pi_i^B = 0$. Under the same conditions,

$$\lim_{d \to b} n_{ss}^C = \frac{(a-\alpha)}{\sqrt{b\beta\rho}} - 1 \,, \tag{23}$$

which is at least as large as two, provided $(a - \alpha) \geq 3\sqrt{b\beta\rho}$.

5 Concluding remarks

We have presented a model where oligopolistic firms compete in a market for differentiated goods, and new varieties are introduced through the innovative activity carried out by an R&D sector operating under perfect competition.

This model has shown that Cournot competition, notoriously milder than Bertrand competition, leaves more resources available for R&D, hence allowing the system to converge to a steady state characterised by a larger number of products as well as firms, compared to Bertrand behaviour. The ultimate consequence is that, contrary to the established wisdom, the social welfare associated with a Bertrand market may well be lower than that associated with a Cournot market.

The conventional result that a higher welfare must be associated to a higher intensity of competition, favouring thus price behaviour against quantity behaviour, stems from a welfare comparison carried out for a given market structure. Removing such limiting hypothesis, it turns out that a harsher market competition translates into a more concentrated market, with negative consequences on social welfare. Our conclusions are in line with analogous points raised by Norman and Thisse (1996) in a static model of spatial competition.

Appendix

The calculation of n_{ss}^C is straightforward. From

$$\pi^C(n) = \frac{(a-\alpha)^2 b}{(2b + d(-1+n))^2} = \rho\beta \tag{a1}$$

we get the following roots:

$$n_-^C = 1 - \frac{2b}{d} - \frac{(a-\alpha)\sqrt{b\beta\rho}}{d\beta\rho} \tag{a2}$$

and

$$n_+^C = 1 - \frac{2b}{d} + \frac{(a-\alpha)\sqrt{b\beta\rho}}{d\beta\rho} \tag{a3}$$

from which it follows immediately that $n_{ss}^C = n_+^C$, since n_-^C is always negative. Moreover, $n_{ss}^C \geq 1$ iff $a - \alpha \geq 2\sqrt{b\beta\rho}$.

Finding n_{ss}^B is more involved, as the equation $\pi^C(n) - \rho\beta = 0$ is cubic in n. Fortunately, only one root is real, so that the remaining two can be disregarded.

As an example, when $\{a = 10; b = 1; L = 100; \alpha = 1/2; \beta = 10; \rho = 1/10\}$, the difference $n_{ss}^C - n_{ss}^B$ is always larger than 3 and increasing in d, for all $d \in [0, 1]$.

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